Atlantic Electronic
Journal of Mathematics
Volume **4**, Number **1**, Winter **2010**

http://aejm.ca
http://aejm.ca/rema
pp. **23-34**

A CLOSER LOOK AT TETRIS: ANALYSIS OF A VARIANT GAME

Kaitlyn M. Tsuruda

Department of Mathematics and Computing Science
Saint Mary's University
Halifax, NS, B3H 3C3, Canada

Abstract. TETRIS is a well-known video game that originated in the 1980s. Despite its popularity and an extensive amount of research on the topic of polyominoes tiling rectangles, many unanswered questions about the mathematical properties of TETRIS remain. For example, if the game is played at a constant speed, does a winning strategy exist such that a perfect player could play indefinitely? This paper outlines the major results surrounding winning strategies that have been developed for TETRIS and presents a variant (non-rectangular) well in which new winning strategies are developed. A set of variant wells are presented, permitting the application of these new strategies.

1. **Introduction.** The game of TETRIS originated in the Soviet Union where it was designed by Alexy Pazhitnov and programmed by Vadim Gerasimov in 1984 [9]. Over 25 years later, it continues to see world-wide success on a multitude of gaming systems and countless variations have been developed [1, 13]. TETRIS has drawn a substantial amount of interest, not only from puzzle enthusiasts, but from psychologists, computer scientists and mathematicians alike [2, 10, 11, 12, 14]. This paper explores the one of the primary questions asked by mathematicians: "Can we win at TETRIS?"

TETRIS is a one player game that is played in a 10×20 rectangular well of empty unit cells (Figure 1). The current piece drop from the centre of the well at a steady speed as the player translates and/or rotates the piece in 90 degree increments until it lands on the bottom of the well or a previously played piece, where it can be moved for an amount of time before "locking" into place. Pieces cannot drop through spaces in which they do not fit. For example, a piece of width two cannot be forced through a gap of width one. The primary objective of the game is to manipulate the pieces to fill the rows. Once all the cells in a given row are full, the row empties (clears) and all rows above it drop by one. The speed at which the pieces falls increases as the game progresses, providing an additional challenge to the player. The game is lost as soon as a cell above row 20 is occupied.

Points are awarded for clearing rows, and both levels and game speed are determined by score: the more points a player has, the higher the level and speed of the game. While some players aim to generate high scores, mathematicians are happy to ignore score and instead focus on how long the game can be played. Certainly if a player can play indefinitely she can make her score as high as she desires!

2000 *Mathematics Subject Classification.* Primary: 05B50, Secondary: 05B40.
Key words and phrases. polyominoes, Tetris, variant well, game.

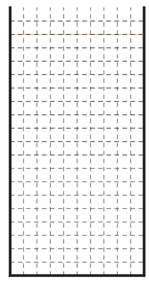

FIGURE 1. An empty Tetris well

TETRIS is played with the reflexive set of Tetrominoes (4 edge-connected unit cells) (Figure 2). This set has 7 members: Bar, Tee, Square, Left Kink, Right Kink, Left Elbow and Right Elbow (sometimes referred to as I, T, O, S, Z, J, and L respectively).

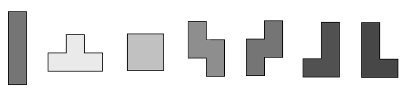

FIGURE 2. The Tetrominoes: Bar, Tee, Square, Left Kink, Right Kink, Left Elbow and Right Elbow (from L to R)

Mathematical models of standard TETRIS make use of a simplified model of the original game. Neither score nor the speed at which the pieces fall will affect a winning strategy and are therefore ignored in the analysis that follows in this paper. (In a game of standard TETRIS, the speed at which the pieces fall increases as one progresses through levels of the game, but if one were to play indefinitely then the speed at which the pieces would fall would eventually be too fast for even the best TETRIS players to succeed. So, we consider a game in which the speed that the current piece drops is as slow as it needs to be for any player to position the pieces where he sees fit.) In this modified model of TETRIS, we say that a player has a winning strategy if he can play indefinitely.

Consider a game of variant TETRIS in a non-rectangular well (Figure 3). The rules of the game in this space should certainly not differ from those in the original well but careful consideration must be taken when clearing rows. If the well is non-rectangular then it may not be possible for every occupied cell in a row above a cleared row to drop. Thus, if is it is possible for occupied cells in a row to drop in the way described in the standard rules, then this rule is followed. Otherwise, the pieces stay stationary until they are cleared from their current row (Figure 4).

2. **Literature.**

The question remains: *does there exist a winning strategy for TETRIS?* John Bruztowski addresses this question in his master's thesis [3]. Further research by Heidi Burgiel [4] expanded on the knowledge base that Bruztowski had developed some years earlier. Both Bruztowski and Burgiel based their research on the standard TETRIS rules previously described.

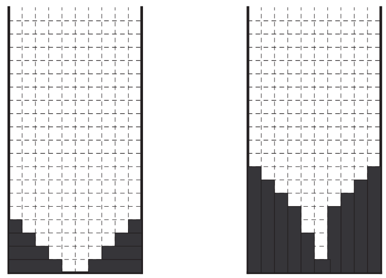

FIGURE 3. Examples of variant wells

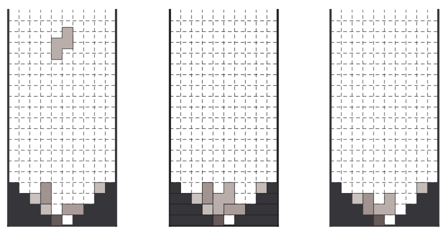

FIGURE 4. A possible clearing sequence in a variant well

2.1. **One Piece Winning Strategies.** The simplest possible game of TETRIS is certainly one in which the computer generates only one piece for the entirety of the game. This leads us to our first theorem:

Theorem 1 (Bruztowski)**.** *Single piece winning strategies exist for all tetrominoes.*

Bruztowski offers a series of state diagrams (images showing the relevant area of the current game) to demonstrate the winning strategies for single piece games. These strategies are straightforward and are similar for all seven Tetrominoes.

Bruztowski extrapolated his single piece winning strategies to hold in rectangular wells of any width but noted that Squares do not have winning strategies for wells of odd width (odd-wells). This result is trivial since Squares are of width two. This result also holds true for Kinks. To show this, Bruztowski introduced the notion of "skew", where skew is the number of full cells in even columns minus the number of full cells in odd columns (columns are numbered from left to right starting at one). A few rotations of the Left and Right Kink pieces demonstrates that the skew is unchanged no matter how a Kink is played. Bruztowski noted that clearing rows in a well of odd-width increases the skew by one for each row cleared. The only way to prevent this is to stop clearing rows, which will ultimately force a loss. On the other hand, to initially force the skew to increase by one would entail increasing the number of even cells in the well which would also force a loss.

2.2. **Two Piece Winning Strategies.** A natural progression from developing one piece winning strategies is to develop winning strategies for two piece combinations. Such strategies will be referred to as "two piece strategies". The development of two piece strategies led to some interesting discoveries by both Bruztowski and Burgiel.

Theorem 2 (Bruztowski). *A game of TETRIS using only the Square and one of Left Elbow, Right Elbow or Bar has a winning strategy*

Proof. Bruztowski divided the columns of the well into lanes (where a lane is two adjacent columns). The 10×20 well has five lanes. The "height" of a lane (or column) is the height of the highest piece played in a lane (or column). A lane is considered to be smooth if both columns have the same height, otherwise it is considered bumpy.

CASE 1 (Square piece): Play the Square in the lowest smooth lane **unless** there is a bumpy lane with a height of at least two less than the other lanes. In this case, the Square is played in the bumpy lane.

CASE 2 (Non-Square piece): If there is a bumpy lane, play the non-Square piece to smooth the lane. If there is no bumpy lane then play the non-Square piece in the lowest smooth lane to make it bumpy.

It's not hard to see that if every lane is smooth then at least one lane is empty. Moreover, there is never more than one bumpy lane, since playing a bumpy piece in the lowest smooth lane requires playing it in an empty lane. □

Bruztowski saw that this strategy was fitting for all rectangular wells of even width. He further developed winning strategies for the random generation of Elbows and Bars as well as for the random generation of Left and Right Elbows using a similar argument. In these cases, two bumpy lanes are required instead of one.

Bruztowski concluded that winning strategies do not exist for all possible pairs of tetrominoes. Specifically, he claimed that if the computer is aware of and can react to your moves, then it can generate a sequence of Left and Right Kinks that will force you to lose. Burgiel took this idea one step further and asserted that almost all TETRIS games will end independent of a reactive game.

Lemma 1 (Burgiel). *In a TETRIS game in which only Kinks are presented to the player, no more than 120 Kinks can be played vertically with their leftmost cells in an even-column or horizontally in any column without losing.*

Proof. Number the columns of the well from $1-10$ (from left to right) and let b_i denote the number of cells played in column i. Let h_i be the number of horizontal Kinks contributing to columns $(i-1)$, (i) and $(i+1)$. Lastly let v_i be the number of vertical Kinks played in columns (i) and $(i+1)$. We notice:

$$b_i = 2v_{i-1} + 2v_i + h_{i-1} + 2h_i + h_{i+1} \tag{1}$$

and we see that $b_i - b_j \leqslant 20$ during game play The game is lost when $b_i - b_j > 20$ since there are necessarily pieces being played outside of the well so we must have $b_i - b_j \leqslant 20$. Further, Kinks cannot be played outside of the well so $h_1 = h_{10} = 0$. We get $b_1 = 2v_1 + h_2 \leqslant 20$, and $b_{10} = 2v_9 + h_9$. We notice that

$$b_2 - b_1 = 2v_2 + h_2 + h_3 \leqslant 20. \tag{2}$$

Likewise, $2v_8 + h_8 + h_9 \leqslant 20$. In general, we have

$$b_{i+1} - b_i = 2v_{i+1} - 2v_{i-1} + h_{i+1} + h_{i+2} - h_i - h_{i-1} \leqslant 20$$
$$\Rightarrow 2v_{i+1} + h_{i+1} + h_{i+2} \leqslant 20 + 2v_{i-1} + h_{i-1} + h_i \tag{3}$$

For $i = 3$, a simple substitution of (2) into (3) gives $2v_4 + h_4 + h_5 \leqslant 40$.
Similarly, $2v_6 + h_6 + h_7 \leqslant 40$. Since $2v_{10} + h_{10} + h_{11} = 0$, we conclude that, in general: $\sum_{i=1}^{10} h_i + \sum_{j=1}^{5} v_{2j} \leqslant 120$. □

Lemma 2 (Burgiel)**.** *The TETRIS game consisting of only Kinks of alternating orientation will always end before* $70,000$ *tetrominoes are played.*

Proof. A player will eventually be forced to play the Kinks vertically with their left-most cells in an odd-numbered column to avoid losing (Lemma 1 shows that he will lose if he does not do so). Once this occurs, we can assume that the topmost Kinks are in non-empty lanes. There are five such lanes and we can assume without loss of generality that there are more Left Kinks than Right Kinks. Hence, until a Right Kink is played in a Left lane, the height of the Right lanes will increase faster than that of the Left lanes. We know that at most 240 Kinks can be played before the player is forced to play a Right Kink in a Left lane (or he will lose). At this time, a hole two cells high and one cell wide will form on the right side of that lane.

The only way such a hole can be filled is to play a Kink vertically between lanes or horizontally, otherwise the hole will remain on the board, unfilled. The game ends when all twenty rows of the well contain such holes, since it will be impossible to fill these holes and clear these rows. A quick calculation shows that it is impossible to have more than 50 holes like this on the board without losing.

Lemma 1 showed that at most 120 moves can be made with vertical Kinks between lanes or horizontal Kinks in any lanes before a game is lost, hence a player can make 120 moves that fill at most two holes per move. Thus, during one game a player might see as many as $50 + (120 \times 2) = 290$ holes, but no more. Since at most 240 Kinks can be played without forming a hole, it is possible to play a total of $240 \times 290 = 69600$ pieces before losing. Thus a game of alternating Kinks will always be lost before 70000 Kinks are played. $\square$

Now a loose upper bound for a specific game of TETRIS has been developed. Unfortunately not all TETRIS games are necessarily finite like the one presented above. A game in which only a single piece is used can be played indefinitely if Bruztowski's single piece strategies are applied. However, Lemmas 1 and 2 can be used to show the following:

Theorem 3 (Burgiel)**.** *Almost all TETRIS games must end.*

Proof. Given a randomly generated infinite string of tetrominoes, one is guaranteed (with probability one) to see a specific finite sequence of pieces. Lemma 2 shows that a game of only alternating Kinks will end before $70,000$ Kinks are played: this is the finite sequence of pieces in which we are interested. Since we are guaranteed to see this sequence of Kinks in an infinite game (i.e. a winning game), we must only show that this sequence of pieces forces a loss for arbitrary initial conditions.

Let a_i be the initial number of cells in column i when we first start the sequence of $70,000$ Kinks. Let b_i be the number of cells added to column i once the string of Kinks starts. Let $d_i = b_i + a_i$ be the total number of cells placed in column i during the entire game. We get:

$$d_i - d_j = a_i - a_j + b_i - b_j \text{ for all } i,\, j.$$
$$\Rightarrow b_i - b_j \leqslant 20 + a_j - a_i.$$
$$\text{And we know that } a_j - a_i \leqslant 20 \Rightarrow b_i - b_j \leqslant 40 \text{ for all } i,\, j.$$

These inequalities are reminiscent of those in Lemma 1. Replacing $b_{i+1} - b_i \leqslant 20$ with $b_{i+1} - b_i \leqslant 40$ in Lemma 1, we see that the bound developed for that particular game is doubled from 120 to 240. Applying the method demonstrated in Lemma 2, we find that a string of $(2 \times 240) + (50 \times 240) = 127,000$ alternating Kinks will force a game with arbitrary initial conditions to end. $\square$

Thus, we must regretfully accept that there does not exist a winning strategy for TETRIS. However, in real time it would take nearly a full hour of play the Kink sequence required to force a loss for a player who can play at a speed of 40 tetrominoes per minute (a relatively quick pace). And so, while we know we are doomed to lose from the start, we don't know how long it will be before the fatal Kink sequence appears, and even then, we are guaranteed that our game will last for a fairly long time even after the end is in sight.

3. **Variant TETRIS.** Above, it was established that there does not exist a winning strategy for a standard game of TETRIS. But perhaps there is a game of TETRIS with a non-rectangular well in which we can play indefinitely. It is no doubt of interest whether there exists a variant well in which a sequence of alternating Left and Right Kinks does not ultimately force a loss.

The variant well used for the analysis presented in this paper is shown in Figure 5. State diagrams illustrate only the bottom eight rows of the well because the outlined strategies utilize only the last seven rows.

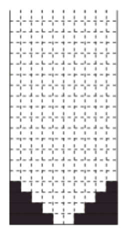

FIGURE 5. Variant well used for analysis of TETRIS in this paper

3.1. **Variant One Piece Winning Strategies.** As with the standard rectangular well, one piece winning strategies exist for all the tetrominoes in the variant well. However, unlike the rectangular wells, the one piece strategies in the variant well are strikingly different from each other, as outlined in the Figures $6-10$. Strategies for Left Kink and Left Elbow are symmetric to those for their corresponding Right counterparts. In all diagrams, "*ci*" indicates that the state clears back to state i.

An interesting but trivial property of these one piece strategies is that they are not unique. While it is not hard to devise various alternate strategies for Bar, Square or either Elbow, it is true that alternate (non-symmetric) strategies exist for both Left and Right Kinks and the Tee as well. An example of an alternate strategy for the Bar is seen in Figure 11.

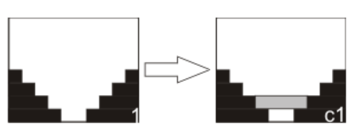

FIGURE 6. A winning strategy for the Bar

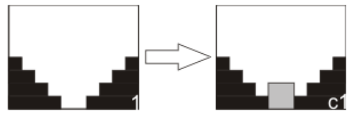

Figure 7. A winning strategy for the Square

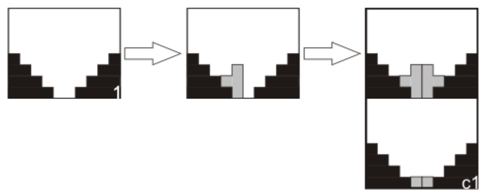

Figure 8. A winning strategy for the Tee

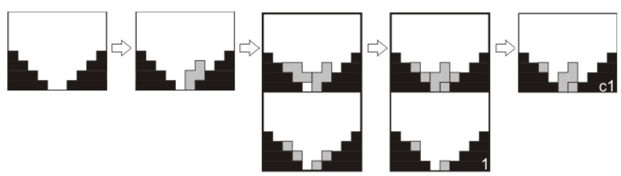

Figure 9. A winning strategy for the Kink

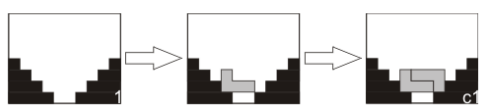

Figure 10. A winning strategy for the Elbow

3.2. **Variant Two Piece Winning Strategies.** Analysis of two piece winning strategies proves to be quite fruitful: winning strategies exist for all 21 possible pairs of tetrominoes when the two pieces are played in an alternating fashion. This section will outline a few of the more interesting strategies.

Perhaps the most obvious way to create a two piece strategy is to "add" the one piece strategies together by following the one piece strategy for the piece that is being played.

The simplest of all two piece strategies is that for Bars and Squares. When alternatively playing these pieces, this method gives a trivial two piece strategy since each piece self clears immediately after being played.

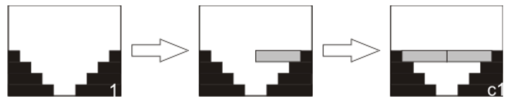

FIGURE 11. An alternate winning strategy for the Bar

The winning strategy for Bar and Tee adds the one piece strategies together, much in the same way that the winning strategy for Square and Bar does. In addition to the one piece strategy presented for the Tee in Figure 8, this strategy also uses as the alternate strategy for Bar presented in Figure 11.

For this strategy to hold, the Bar must be the first piece played. If the Tee is the first piece given then it should be played vertically in column one or ten. When a Bar is played the given alternate strategy can be employed, that is, the Bar is played horizontally with its rightmost cell in column nine. The proceeding Tees and Bars can be played according to their respective standard one piece strategies. Careful attention must be paid when playing the Tees because the first Tee needs to be played underneath the previously played Bar. For this to happen it must be translated horizontally to the right twice: once after being initially dropped in column four and again after dropping into column five. Playing in this manner will ensure a winning strategy for the player since the well completely clears after at most five pieces are played (with the possible exception of the initial Tee).

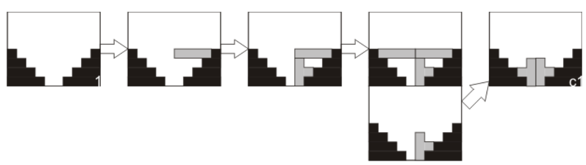

FIGURE 12. A winning strategy for alternating Bars and Tees

Unlike in the rectangular well, there exists a winning strategy for alternating Kinks of different orientation in the presented variant well. The strategy is surprisingly straightforward: playing the Right Kinks just to the left of the centre of the well and the Left Kinks just to the right of centre of the well insures a winning strategy (figure 13).

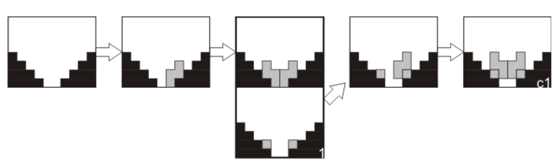

FIGURE 13. A winning strategy for alternating Left and Right Kinks

Unfortunately, not all two piece strategies in the variant well are straightforward. The strategy shown below for Kinks and Bars (figure 14) is certainly non-trivial.

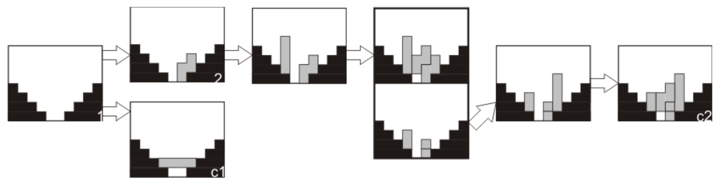

FIGURE 14. A winning strategy for alternating Kinks and Bars

3.3. **Random Generation of Pieces.** So far, there has been no indication that there does not exist a winning strategy for TETRIS in the variant well: single piece strategies exist for all the tetrominoes and two piece strategies exist for all pairs of tetrominoes as long as they alternate in game play. It is also true that winning strategies exist for the random generation of certain combinations of pieces. Such strategies will be referred to as general strategies.

It is obvious that the two piece strategy presented for the Bar and Square also serves as a general strategy. And, just as there exists a general strategy for Bars and Elbows in a standard well, there also exists a general strategy for these pieces in the variant well. Not surprisingly, this strategy is completely different from that in the standard well.

Theorem 4. *A game of variant TETRIS using only the Bar and either the Left Elbow or the Right Elbow has a general winning strategy.*

Proof. Consider the four centre columns of the well (that is, columns 4-7 inclusive) as the "centre". We will call columns three and eight the inner columns and columns two and nine the outer columns. Columns one and ten are not used in this strategy. Here, an empty column is a column with no occupied cells and full column is a column with more than one occupied cell. A column with only one occupied cell is an intermediate column. The two cases for this strategy depend on the current piece being played.

<u>*CASE 1. Elbow:*</u> Elbows are always played horizontally in the centre using the presented one piece strategy (as in Figure 10).

<u>*CASE 2. Bar:*</u> There are two subcases regarding the placement of the Bar piece: *Case 2a. The centre is empty:* Here, the Bar is played horizontally in the centre so that it self clears (as in Figure 6).

Case 2b. The centre is not empty: Here, there is either an empty (or intermediate) inner column or the inner columns are full and there is an empty outer column or both the inner and outer columns are full. If there is an empty (or intermediate) inner column then the Bar is played vertically in the empty (or intermediate) inner column. Similarly, if the inner columns are full but there is an empty outer column then the Bar should be played vertically in either empty outer columns. If both of the inner and outer columns are full then the Bar should be played horizontally in the centre. A Bar must never be played on top of another Bar in full inner or outer columns, this type of move is reserved only for the intermediate columns. Notice that the horizontal Bar will always clear since if there is an Elbow in the centre,

the horizontal Bar is in row four and clears along with one cell from each of the inner and outer columns. If there are no Elbows in the centre then this is Case 2a. Figure 10 demonstrates why there can never be two Elbows in the centre of the well when this strategy is followed. □

4. **Extentions and Other Variant Wells.** The strategies presented for the game of TETRIS in the variant well can be used in a variety of different wells. Careful consideration of the previously presented strategies shows that they all take place in a limited, central area of the well as highlighted below in grey (Figure 15). A set of variant wells in which the presented strategies can be applied can be extrapolated using the shape of this space (some examples are shown in Figure 16). While these wells must have finite size, in some cases this size may be quite large. Here it will be useful to define the base height, h_{b_i}, of column i as the first row in which it is possible to play a piece.

FIGURE 15. The central area utilized by the strategies presented in this paper

All variant wells must necessarily be of even width, otherwise it would take no more than a sufficiently long sequence of Squares to force a loss. Further, all variant wells must necessarily be at least ten cells wide and eight high or some of the strategies will no longer hold true.

Wells in which there is only one area to play are standard in TETRIS. For variant wells with this property, columns one and ten may be of any height so long as there is sufficient space in these columns to play a vertical Bar. That is, h_{b_1}, $h_{b_{10}} \leqslant h_{tot} - 4$ where h_{tot} is the total height of the board. If columns one and ten were any higher than this, it would be impossible to tuck a tetromino out of the required playing area in the two piece alternating games. Also note that the two centre columns (columns five and six) may be any height so long as h_{b_5}, $h_{b_6} < h_{b_4} = h_{b_7}$. In variant boards the height of the centre two columns need not be the same, that is h_{b_5} needn't necessarily equal h_{b_6}. That said, it is essential that h_{b_2} and h_{b_9} are one higher than h_{b_3} and h_{b_8}, which need be one higher than h_{b_4} and h_{b_7}. It is also necessary that $h_{b_2} = h_{b_9}$, $h_{b_3} = h_{b_8}$ and $h_{b_4} = h_{b_7}$.

It is also possible to create a variant well in which there are essentially two areas to play (Figure 17). These wells must have the same properties as the variant wells described above. It is important that the columns in the lower area have enough playable area before the commencement of the higher playing area. As long as the lower area has sufficient free space, the higher area is free to have an arbitrary design and vice versa. It is also possible to have a well with only one area but that is essentially closed at the top for if sufficient space is given in the lower area of the well then area at the top is superfluous.

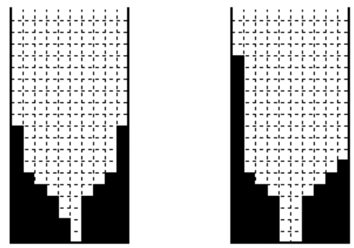

FIGURE 16. Examples of wells with a single playing area

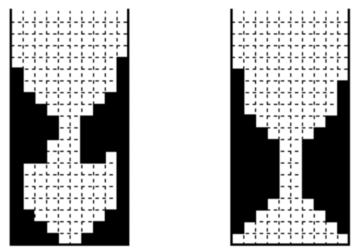

FIGURE 17. Examples of wells with two playing areas

In wells with multiple playing areas, it is important that the game is played in the area that matches the description of the wells with only a single playing area since the presented strategies may fail otherwise. Further, it is important to not block off the bottom area by a misplaced piece in the top area if game play is occurring in the lower space.

5. **Conclusions and Open Problems.** Similar to the rectangular well, one piece winning strategies exist for all the tetrominoes in the presented variant wells. Two piece winning strategies exist for all pairs of tetrominoes as long as they are played in an alternating fashion and general winning strategies exist for the random generation of certain pairs of tetrominoes. However, it remains unknown whether TETRIS in this variant well can (almost) always be played indefinitely. It is likely that there exists a sequence of pieces that can force a loss in this variant well but such a sequence remains unknown. While the general strategy for Bars and Elbows is simple, it requires over 100 state diagrams to check all the cases. This is perhaps an indication of the complexity of the problem [2, 10].

Although it remains unknown whether there exist a general winning strategy for TETRIS in the presented variant wells, it would likely be acceptable to make a variant game using any of the alternative wells with one playing area presented in this paper due to the multitude of other winning strategies that exist in these wells. The challenge posed by such a variation in the game would be comparable to the challenge presented in regular TETRIS and, given the success of the original game, one would hope to see similar success with this variant game as well. Unfortunately, the wells presented in section 4 with more than one playing area may not have enough free space to provide an enjoyable game with random generation in real-time. Inevitably a human player will make mistakes and there is not much free space in which mistakes can be "corrected" in these wells; without careful thought, a game could end after very few pieces have been played.

An open problem suggested by Burgiel in her paper "How to lose at TETRIS" is to find a more accurate lower bound on the number of alternating Kinks played before a loss in a standard game TETRIS. She estimates that this bound is closer to 1300 than her demonstrated $130,000$. Further, Burgiel suggests that it is possible that alternating Kinks are not the only sequence of tetrominoes that force a game of TETRIS to end.

The set of reflexive pentominoes (five edge-connected cells) has 18 members; there is no doubt that a game played with pentominoes is more difficult than one played with the tetrominoes. Does the game change drastically if pentominoes are used in place of tetrominoes? The P-Pentomino has a trivial winning strategy while the X-Pentomino cannot fill two successive rows (even with row clearing).

Lastly, It would also be interesting to see how many other variant wells can be developed that permit the application of the presented winning strategies. How many of these variant wells are not symmetric? Which, if any, combinations of pieces have general strategies? Square and Elbow have a relatively easy general strategy in the standard well but a general strategy for these pieces in variant wells is not trivial. The question remains at large: can we win in the presented version of TETRIS, or does there exist a sequence of tetrominoes that force us to lose?

Acknowledgements. The author would like to extend her sincere gratitude to Dr. R. Dawson for his guidance and support with this project.

REFERENCES

[1] D. Baccherini, D. Merlini, *Combinatorial analysis of Tetris-like games*, Discrete Math., **308(10)**, 4165–4176.

[2] R. Bruekelaar, E. D. Demaine, S. Hohenberger, et al., emphTetris is hard, even to approximate, Int. J. Comput. Geom. Ap., **14(1/2)**, 41–68.

[3] J. Bruztowski, *Can You Win at Tetris?*, Masters' Thesis, The University of British Columbia (March 1992).

[4] H. Burgiel, *How to Lose at Tetris*, The Math. Gazette, **81(491)** (July 1997), 194–200.

[5] B. Grünbaum and G. C.Shephard, Tilings and Patterns, W. H. Freeman and Company, New York (1987) 26–34, 497–510.

[6] D. W. Walkup, *Covering a rectangle with T-tetrominoes*, Amer. Math. Monthly, **72** (1965), 968–988.

[7] G. E. Martin, Polyominoes: A guide to puzzles and problems in tiling, The Math. Assoc. of Amer., (1991).

[8] *"Henk Rogers"*, Blue Planet Software. Accessed 23 Feb. 2009
`<http://main.blueplanetsoftware.com/biography_henk_rogers.html>`.

[9] *"History"*, Tetris. 2006. Accessed 9 Sept. 2008
`<www.tetris.com>`.

[10] H. J. Hoogeboom, W. A. Kosters, *Tetris and decidability*, Information Processing Letters, **89(6)** (March 2004), 267–72.

[11] K. Laczika, T. Staudinger, G. J. Locker, et al., *Tetris and physician's performance state*, The Lancet, **346(8973)** (August 1995) 516.

[12] B. Loguidice, M. Barton, Tetris (1985): Casual Gaming Falls into Place. Vintage Games. Focal Press, Boston (2009) 291–301.

[13] C. Mandel, *'Congratulations, Player No. 1...'*, Canadian Business, **70(13)** (1997) 114.

[14] M. K. Sims, R. E. Mayer, *Domain specificity of spatial expertise: the case of video game players*, Appl. Cognit. Psychol., **16(1)** (2002), 97–115.

E-mail address: `k.tsuruda@gmail.com`